# Unit 1 Integers and decimals

## Integers

All whole numbers are called integers.
Integers can be positive or negative.
Zero is an integer.

Remember…
When you move left on a number line, numbers get smaller, when you move right on a number line, numbers get larger.

$\leq$ means 'less than or equal to'
$\geq$ means 'greater than or equal to'

**1** To which number does each arrow point?



**2** Look at the number line above. Write the difference between these numbers.

**a)** a and c　　　　**b)** d and e　　　　**c)** b and f　　　　**d)** e and a

**3** Which integers could go in the boxes?

**a)** $-4 < \boxed{\phantom{x}} < 0$　　**b)** $-11 < \boxed{\phantom{x}} < -8$　　**c)** $-3 < \boxed{\phantom{x}} < 2$　　**d)** $-21 < \boxed{\phantom{x}} < -17$

**e)** $-9 > \boxed{\phantom{x}} > -12$　　**f)** $-1 > \boxed{\phantom{x}} > -6$　　**g)** $-5 > \boxed{\phantom{x}} > -9$　　**h)** $-19 > \boxed{\phantom{x}} > -23$

**4** Which integers could go in the boxes?

**a)** $-7 \leq \boxed{\phantom{x}} \leq -2$　　**b)** $-1 \leq \boxed{\phantom{x}} \leq 4$　　**c)** $-14 \leq \boxed{\phantom{x}} \leq -8$　　**d)** $-6 \leq \boxed{\phantom{x}} \leq -1$

**e)** $0 \geq \boxed{\phantom{x}} \geq -5$　　**f)** $-2 \geq \boxed{\phantom{x}} \geq -4$　　**g)** $3 \geq \boxed{\phantom{x}} \geq -1$　　**h)** $-15 \geq \boxed{\phantom{x}} \geq -19$

**5** What is the difference in temperature between these pairs of thermometers?

**a)**
−70 −60 −50 −40 −30 −20 −10  0  10  20  30  40  50  60  70  80  90  100 °C

−70 −60 −50 −40 −30 −20 −10  0  10  20  30  40  50  60  70  80  90  100 °C

**b)**
−70 −60 −50 −40 −30 −20 −10  0  10  20  30  40  50  60  70  80  90  100 °C

−70 −60 −50 −40 −30 −20 −10  0  10  20  30  40  50  60  70  80  90  100 °C

**c)**
−70 −60 −50 −40 −30 −20 −10  0  10  20  30  40  50  60  70  80  90  100 °C

−70 −60 −50 −40 −30 −20 −10  0  10  20  30  40  50  60  70  80  90  100 °C

**d)**
−70 −60 −50 −40 −30 −20 −10  0  10  20  30  40  50  60  70  80  90  100 °C

−70 −60 −50 −40 −30 −20 −10  0  10  20  30  40  50  60  70  80  90  100 °C
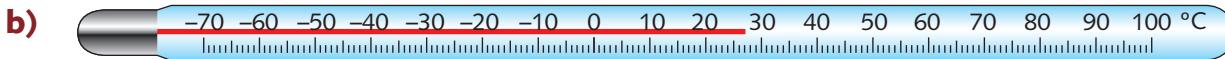
**6** Write these temperatures in order, starting with the lowest.

| 38° | −7° | −14° | 0° | 27° | −24° |

## Try this

Jack was trying to throw a coin exactly 2 metres. He recorded each attempt in centimetres above or below his target.

| Attempt | 1st | 2nd | 3rd | 4th | 5th | 6th | 7th | 8th |
|---|---|---|---|---|---|---|---|---|
| Distance from target (cm) | +3 | +5 | −3 | −7 | −1 | 0 | +1 | −2 |

**a)** What was his longest throw in centimetres?
**b)** What was his shortest throw in centimetres?
**c)** On which attempt did he hit the target?
**d)** On which attempt did he throw 197 cm?
**e)** How would he have recorded a throw of 192 cm?

# Rounding and approximation

When working with large numbers, rounding makes them easier to work with.

Remember…
Rounding means changing a number to the nearest 10, 100, 1000, 10 000 or 100 000.

**Example**

| Number 48 193 065 | nearest 10 48 193 070 | nearest 100 48 193 100 | nearest 1000 48 193 000 | nearest 10 000 48 190 000 |
|---|---|---|---|---|

**1** Copy and complete this table.

| | **a)** Round to the nearest 100 | **b)** Round to the nearest 1000 | **c)** Round to the nearest 10 000 |
|---|---|---|---|
| 7 892 388 → | | | |
| 68 372 105 → | | | |
| 38 893 465 → | | | |
| 149 035 476 → | | | |
| 7 498 024 573 → | | | |
| 1 093 773 284 → | | | |
| 1 936 243 225 → | | | |
| 7 846 374 522 → | | | |

**2** Write the smallest and largest numbers that will give the following.
a) 8 460 000 when rounded to the nearest ten thousand.
b) 74 110 000 when rounded to the nearest ten thousand
c) 397 500 000 when rounded to the nearest hundred thousand
d) 649 900 000 when rounded to the nearest hundred thousand

**3** Round these numbers.
Decide on the type of rounding to use so that the number you get has just one digit followed by zeros.
a) 44 618　　b) 256 700　　c) 12 054 000　　d) 164 000
e) 11 162　　f) 5 602 721　　g) 3 532 000　　h) 212 500 000

**4** Round these distances of the planets from the Sun to the nearest ten thousand, hundred thousand or million.
Decide which one to round to so that the information is still sensible and useful.

| Planet | Distance from Sun (km) |
|---|---|
| Mercury | 57 918 438 |
| Venus | 108 238 629 |
| Earth | 149 621 403 |
| Mars | 227 918 304 |
| Jupiter | 778 324 941 |
| Saturn | 1 427 030 429 |
| Uranus | 2 871 302 704 |
| Neptune | 4 497 104 396 |
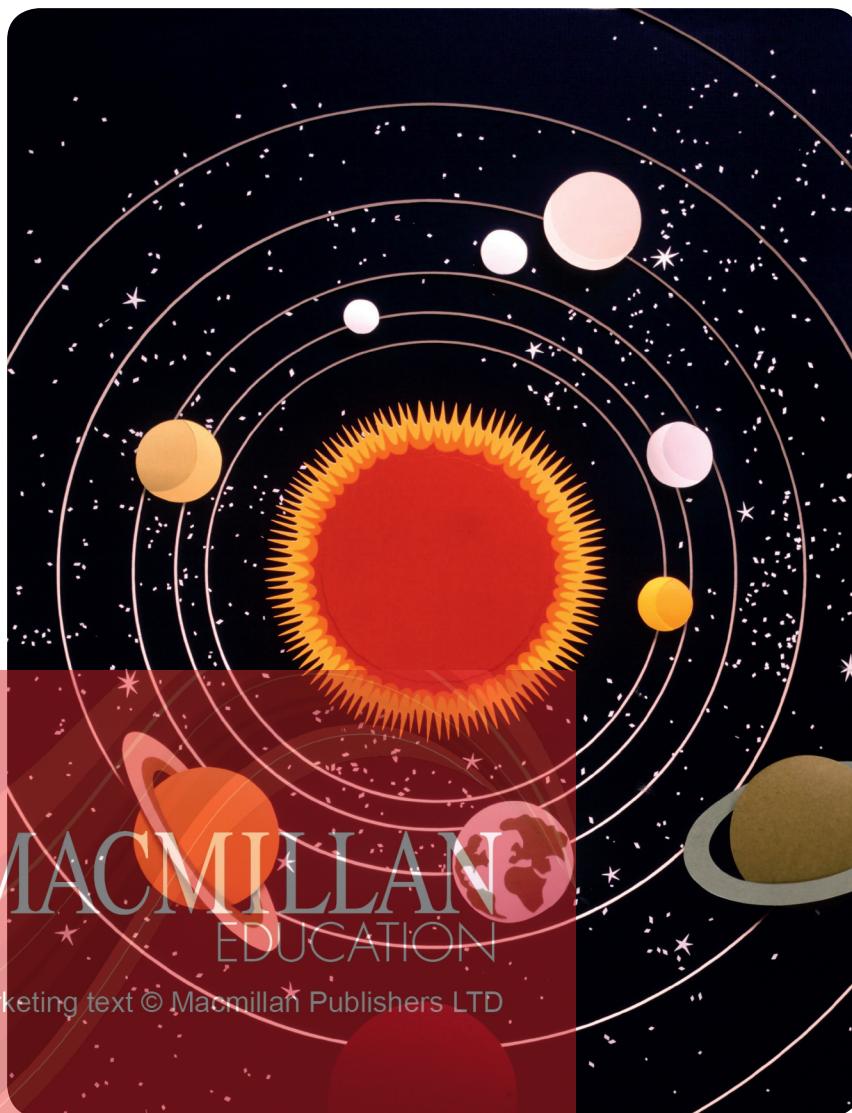
## Try this

**a)** How many numbers give 7 000 000 when they are rounded to the nearest thousand?

**b)** How many numbers give 7 000 000 when they are rounded to the nearest ten thousand?

**c)** How many numbers give 7 000 000 when they are rounded to the nearest hundred thousand?

**d)** How many numbers give 7 000 000 when they are rounded to the nearest million?

**e)** Do you get the same results if you choose a different rounded value? Try it for 12 000 000.

**f)** Can you make any predictions using these results?

# Large numbers

Mathematicians often use abbreviations called **index form** to write large numbers in a shorter way. They use powers of 10 to show the number of zeros.

$10 \times 10 = 10^2$              $10 \times 10 \times 10 = 10^3$              $10 \times 10 \times 10 \times 10 = 10^4$
$100 = 10^2$                              $1000 = 10^3$                              $10\,000 = 10^4$

This is how large numbers are written:

$8400 = 84 \times 10^2$              $129\,000 = 129 \times 10^3$              $650\,000 = 65 \times 10^4$

**Did you know?**

One billion means one thousand million.
$1\,000\,000\,000 = 10^9$

An American invented the name googol for the number $10^{100}$.

**1** Write these numbers in full.

a) $67 \times 10^2$              b) $5 \times 10^4$              c) $85 \times 10^3$

d) $23 \times 10^4$              e) $38 \times 10^5$              f) $162 \times 10^3$

g) $15 \times 10^6$              h) $32 \times 10^4$              i) $12 \times 10^5$

j) $11 \times 10^3$              k) $294 \times 10^4$              l) $2 \times 10^8$

**2** Write these in index form.

a) 26 000              b) 30 000              c) 294 000

d) 1 800 000              e) 61 000 000              f) 70 000 000

g) 3 810 000              h) 292 000 000              i) 270 000 000

j) 300 000 000              k) 22 000 000              l) 4 830 000 000

**3** Copy these sentences, replacing the numbers using index form.

**a)** The Milky Way is about 100 000 light years across.

**b)** Astronomers think that there are approximately 200 000 000 000 000 000 000 stars.

**c)** Some stars have a diameter of more than 150 000 000 kilometres.

**d)** The Sun is approximately 149 000 000 kilometres from Earth.

**e)** The temperature in the middle of the Sun is approximately 15 000 000°C.

## Try this

**a)** Multiply these two numbers together.

$$10^3 \times 10^4$$

Convert them to full numbers first, then multiply them.

**b)** Convert the answer into index form.

Do you notice a connection between the answer and the original numbers?

**c)** Multiply these two numbers together.

$$(2 \times 10^5) \times (4 \times 10^3)$$

Convert them to full numbers first, then multiply them.

**d)** Convert the answer into index form.

Do you notice a connection between the answer and the original numbers?

Investigate this with some of your own index form multiplications.

# Decimal numbers

The decimal point separates whole numbers from decimal fractions.

| tens | ones | | tenths | hundredths | thousandths | ten thousandths |
|---|---|---|---|---|---|---|
| 3 | 8 | . | 4 | 1 | 5 | 2 |
| (30) | (8) | | $(\frac{4}{10})$ | $(\frac{1}{100})$ | $(\frac{5}{1000})$ | $(\frac{2}{10000})$ |

38.4152 is read as **thirty-eight point four one five two**.

The value of the digit 2 is 2 ten-thousandths or $\frac{2}{10000}$, which is a very small fraction!

Decimals are usually rounded to the nearest whole number or nearest tenth.

| **Rounding to the nearest whole number** | **Rounding to the nearest tenth** |
|---|---|
| • Look at the tenths digit. | • Look at the hundredths digit. |
| • If it is 5 or more, round up to the next whole number. | • If it is 5 or more, round up to the next tenth. |
| • If it is less than 5, the units digit stays the same. | • If it is less than 5, the tenth digit stays the same. |
| 18.6209 rounds up to 19 | 18.5627 rounds up to 18.6 |
| 3.3948 rounds down to 3 | 11.9139 rounds down to 11.9 |

**1** Write the decimal number each arrow points to.

a)



b)

c)

d)
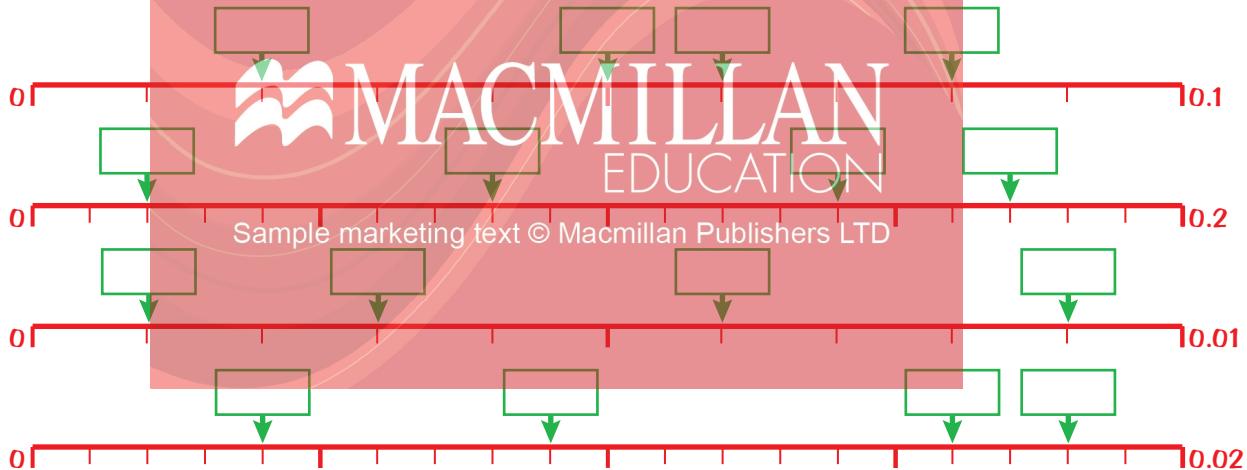
**2** Read the decimal numbers from question 1 and write each one in words.

**3** Write each set in order, starting with the smallest.

a)
```
19.407
19.74
19.007
19.9
```

b)
```
0.0035
0.033
0.302
0.3302
```

c)
```
6.445
6.5034
6.3559
6.4412
```

d)
```
30.9312
30.9132
30.0913
30.1903
```

**4** Round each amount to the nearest whole number.

   **a)** 61.39 cm     ➜         **b)** 8.085 litres     ➜

   **c)** $315.45     ➜         **d)** 35.285 g     ➜

   **e)** 19.62 km     ➜         **f)** 18.096 kg     ➜

**5** Round each amount to the nearest tenth.

   **a)** $36.45     ➜         **b)** 8.214 litres     ➜

   **c)** 37.492 m     ➜         **d)** 26.743 kg     ➜

   **e)** 134.264 km     ➜         **f)** $37.62     ➜

**6** These are the lengths and weights of some of the smallest mammals in the world.

| Mammal | Length (m) | Weight (kg) |
| --- | --- | --- |
| African pygmy mouse | 0.062 | 0.0081 |
| Asiatic shrew | 0.074 | 0.0054 |
| Birch mouse | 0.073 | 0.0109 |
| Desert shrew | 0.058 | 0.0038 |
| Pygmy shrew | 0.039 | 0.0025 |
| White-toothed shrew | 0.081 | 0.0113 |

   **a)** Write the mammals in order of length, starting with the shortest.

   **b)** Write the mammals in order of weight, starting with the lightest.

   **c)** Round each length to the nearest millimetre.

   **d)** Round each weight to the nearest gram.

**Try this**

   **a)** What number does 3.8025 have to be multiplied by to get 380.25?

   **b)** What number does 518.22 have to be divided by to get 51.822?

   **c)** A number is multiplied by 1000 to give 2.1. What is the number?

   **d)** What number divided by 100 gives 3.0418?

   **e)** A number is divided by 1000 to give 3.610 25. What is the number?

   **f)** A number is multiplied by 1000 to give 29.03. What is the number?

# Adding and subtracting decimals

When you add and subtract, estimate an approximate answer first.
To find an approximate answer, round to the nearest 10 or 1 to make the numbers easy to calculate in your head.

| **Example 1** | **Example 2** |
|---|---|
| What is 364.74 added to 107.49? | What is 4.651 subtract 1.965? |
| An approximate answer is 360 + 110 = 470 | An approximate answer is 5 – 2 = 3 |
| $3^1\,6^1\,4\,.^1\,7\,4$ <br> $+\;1\;\;0\;\;7\;.\;4\;\;9$ <br> $\underline{\quad4\;7\;2\;.\;2\;3\quad}$ | $^3\,4\,.^{15}\,6^{14}\,5\,^1\,1$ <br> $-\;1\;.\;9\;\;6\;\;5$ <br> $\underline{\quad2\;.\;6\;\;8\;\;6\quad}$ |

**1** Write approximate answers as whole numbers, then calculate the exact answer.

a) $\quad5\,.\,6\,5\,8$
$\underline{+\,2\,.\,7\,5\,2}$

b) $\quad1\,3\,.\,2\,7$
$\underline{+\,5\,1\,.\,8\,2}$

c) $\quad5\,.\,9\,0\,3$
$\underline{+\,2\,.\,3\,1\,9}$

d) $\quad4\,1\,2\,.\,7\,9$
$\underline{+\,1\,7\,8\,.\,1\,6}$

e) $\quad6\,1\,.\,5\,8$
$\underline{-\,3\,9\,.\,5\,2}$

f) $\quad4\,9\,6\,.\,9\,1$
$\underline{-\,2\,0\,8\,.\,9\,6}$

g) $\quad9\,.\,4\,1\,7$
$\underline{-\,7\,.\,2\,9\,8}$

h) $\quad3\,0\,.\,4\,2$
$\underline{-\,1\,9\,.\,7\,8}$

**2** Read and answer these. Write an approximate answer and an exact answer.

a) Add 29.08 to 38.44.

b) What is the sum of 235.88 and 129.26?

c) Total 1.717 and 4.355.

d) What is 8.794 subtract 5.097?

e) What is the difference between 700.63 and 291.44?

f) What is 26.35 less than 56.183?

**A**

3.245 kg

**B**

11.88 kg

**C**

2.915 kg

**D**

4.203 kg

**E**

12.79 kg

**F**

16.24 kg
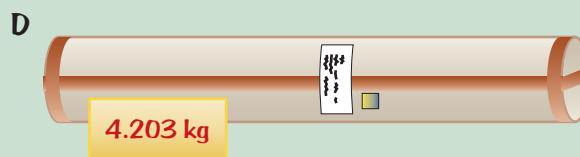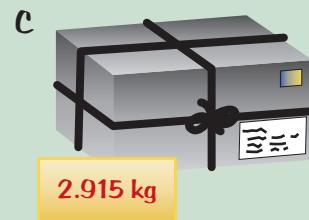
**1** Write the parcels in order of weight, starting with the heaviest.

**2** Round each weight to the nearest kilogram.

**3** Round each weight to the nearest tenth of a kilogram.

**4** Answer these.

    **a)** What is the total weight of parcels C and D?

    **b)** How much do parcel B and E weigh altogether?

    **c)** Parcel A and parcel C are carried together. What is the total weight being carried?

    **d)** What is the difference in weight between parcels F and B?

    **e)** How much more does parcel D weigh than parcel A?

    **f)** How much less does parcel E weigh than parcel F?

**5** Answer these.

    **a)** Which two parcels have a total weight less than 7 kg?

    **b)** What is the total weight of parcel B, parcel E and parcel F? Write both the approximate weight and the exact weight.

    **c)** Which parcel weighs 0.33 kg less than parcel A?

    **d)** Which two parcels have a difference in weight of 0.91 kg?